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GEOMETRICAL INFLUENCES ON THE RADIANT HEAT TRANSFER WITHIN AN ENCLOSED SPACE

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Abstract—In engineering practice, the heat transfer rate by radiation from hot objects to their surroundings is usually calculated with the equation:

$$\phi_s = A_1\sigma(T_1^4 - T_2^4)\left[\frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right)\right]^{-1}.$$

However, this equation is exactly valid only if the arrangement of the surfaces A_1 and A_2 is completely symmetrical.

In this investigation the radiant transport was measured between a heated strip and a slender, cooled cylinder, the strip width and the distance of the strip to the cylinder wall being varied.

In these situations, even though highly asymmetrical, the radiant transport was predicted with acceptable engineering accuracy by the above equation.

NOMENCLATURE

A_1, A_2, surface area [m²];
a_1, a_2, absorption (emission) coefficient;
b, width of the strip [m];
r, radius of the cylinder [m];
s, distance from a point on surface 1 to a point on surface 2 [m];
T_1, T_2, temperatures [°K];
β_1, β_2, angle with the normal to the surface 1(2);
ε, measure of the eccentricity of the strip in the cylinder (see Fig. 2);
σ, radiation constant $= 5{\cdot}75 \times 10^{-8}$ [W/m² °K⁴];
ϕ_s, net radiation transport [W];
$\phi''_{s,1\to2}$, $\phi''_{s,2\to1}$, heat flux by radiation per unit surface area which arrives at a given point of the surface 2(1) and which comes from the total surface 1(2) [W/m²];
ϕ''_{s1}, ϕ''_{s2}, total heat flux by radiation per unit surface area which is emitted from surface 1(2) at a given point [W/m²].

INTRODUCTION

THE CALCULATION of the heat transfer by radiation involves simultaneous solution of two integral equations. Even with a simple geometry the mathematics are quite complex. Frequently, therefore, an approximate solution to the problem is accepted. The foundation of this approximation will be explained below.

The net radiation transport ϕ_s between two gray surfaces A_1 and A_2 each with uniform radiation properties and temperatures T_1 and T_2 (see Fig. 1), is found as the solution to the following system of equations:

$$\phi''_{s,1\to2} = \int_{A_1} \phi''_{s1}\frac{\cos\beta_1\cos\beta_2}{\pi s^2}\,\mathrm{d}A_1, \quad (1)$$

$$\phi''_{s,2\to1} = \int_{A_2} \phi''_{s2}\frac{\cos\beta_1\cos\beta_2}{\pi s^2}\,\mathrm{d}A_2, \quad (2)$$

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$$\phi''_{s1} = a_1\sigma T_1^4 + (1 - a_1)\phi''_{s,2\to 1}, \tag{3}$$

$$\phi''_{s2} = a_2\sigma T_2^4 + (1 - a_2)\phi''_{s,1\to 2}, \tag{4}$$

$$\phi_s = \int_{A_2} \phi''_{s,1\to 2}\,\mathrm{d}A_2 - \int_{A_1} \phi''_{s,2\to 1}\,\mathrm{d}A_1. \tag{5}$$

Equations (1) and (2) represent two simultaneous integral equations, in which $\phi''_{s,1\to 2}$ and $\phi''_{s,2\to 1}$ are the unknown functions. $\phi''_{s,1\to 2}$ is the heat flux per unit surface which arrives at a given point in the surface A_2 and which came from the total surface A_1. ϕ''_{s1} is the total heat flux per unit surface which departs from the surface A_1 at a given point. ϕ''_{s1} consists of two components, viz. the emission from the surface A_1 itself and the reflection of a portion of the radiation coming from surface A_2. The group $(\cos\beta_1 \cos\beta_2/\pi s^2)$ in the integral equations represents the fraction of the radiation emitted from a surface element $\mathrm{d}A_1$ which arrives at a surface element $\mathrm{d}A_2$ and vice versa (see Fig. 1).

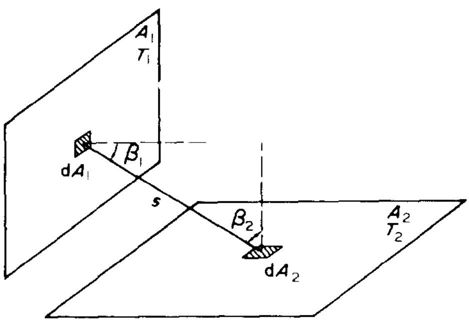

FIG. 1. Illustration of the variables in equations (1)–(5).

The simplification which may now be applied, is that the unknown functions $\phi''_{s,2\to 1}$ and $\phi''_{s,1\to 2}$ may be replaced by the values obtained by averaging the functions over the surface A_1 and A_2. Then the integrals in the equations (1) and (2) are dependent upon the geometry only. For each arrangement of the surfaces A_1 and A_2 these integrals can be calculated in advance. The system of simultaneous integral equations is hereby reduced to a system of linear, algebraic equations, which is much easier to solve. In this investigation the simplified theory was checked experimentally in order to gain some insight into the nature of the errors which are introduced with the approximation.

The geometry was chosen such that the surface A_1 was enclosed by the surface A_2. The simplified model predicts the net radiation transport between A_1 and A_2:

$$\phi_s = A_1\sigma(T_1^4 - T_2^4)\left[\frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right)\right]^{-1}. \tag{6}$$

This is the relation which is frequently employed in technical work to calculate radiation transfer from hot objects to their cold surroundings. Often moreover the surface A_1 is much smaller than the surface A_2 so that equation (6) may be reduced to:

$$\phi_s = A_1 a_1 \sigma(T_1^4 - T_2^4). \tag{6a}$$

It is remarkable that according to equation (6) the net radiation transport is independent of the geometry: only the size of the two surfaces is of importance. In this investigation surface A_1 is a metal strip and surface A_2 a slender cylinder (see Fig. 2).

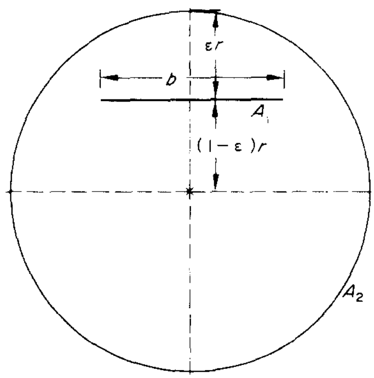

FIG. 2. Topview of the arrangement of cylinder and strip.

With a fully symmetrical geometry, e.g. if A_1 and A_2 are the surfaces of two concentric, infinitely long, circular cylinders, then equation (6) is at the same time the exact solution of the system of equations (1)–(5). In this case $\phi''_{s,2\to 1}$

and $\phi''_{s,1\to2}$ are uniform on the surface A_1 and A_2, respectively. If the geometry is not completely symmetrical, e.g. a strip in a circular cylinder, equation (6) is no longer expected to predict the net transfer. Now the resolution of the system of equations (1)–(5) will be of the following form:

$$\phi_s = A_1 \sigma (T_1^4 - T_2^4) Q^{-1}, \tag{7}$$

where Q is a factor which depends only upon the geometry and the radiation properties. If the geometry is fully symmetrical then, according to equation (6), the factor Q must be:

$$Q = \frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right). \tag{8}$$

The purpose of the experiments was to determine how much the true value of Q in the case of a strip in a cylinder deviates from the value predicted by equation (8).

EXPERIMENTAL

The principle of the measurements is as follows: a metal strip heated electrically is placed in an evacuated circular cylinder in the manner shown in Fig. 2. For various widths of the strip and various values of the eccentricity (see Table 1), the heating power of the strip was measured as a function of the temperature of the strip T_1, the temperature of the cylinder T_2 being held as nearly constant as possible. The heating power is equal to the heat transfer by radiation. Thus with the aid of equation (7) the quantity Q can be calculated.

Table 1. Data on the experiments performed

Strip width (mm)	A_1 (mm^2)	A_1/A_2
2·6	$1{\cdot}04 \times 10^3$	0·04
5·2	$2{\cdot}14 \times 10^3$	0·08
10·0	$4{\cdot}07 \times 10^3$	0·15
14·9	$6{\cdot}75 \times 10^3$	0·225
19·8	$8{\cdot}12 \times 10^3$	0·300
24·5	$9{\cdot}95 \times 10^3$	0·370
35·0	$14{\cdot}2 \times 10^3$	0·525

Inside diameter of cylinder: 42 mm. Length of cylinder: 450 mm, ε = 0·96; 0·90; 0·75; 0·60; 0·45; 0·30, T_1 = 300°C; 400°C; 500°C.

The cylinder was made from a stainless steel tube which was honed on the inside in order to obtain a "dull gleaming" surface. This was done to assure diffuse reflection. The cylinder is so slender that it can be considered infinitely long. A cooling jacket was placed around the cylinder. With the aid of cooling water the cylinder is kept at constant temperature T_2. This temperature varied a little from measurement to measurement due to fluctuations in cooling water temperature: the minimum value of T_2 was 5°C, the maximum 18°C. Because T_1 is very much higher than T_2 the influence of this variation on the value of Q is however very small. The cylinder was closed tightly by two cover-plates which were insulated electrically from the cylinder. The heating current was supplied to the strip via the covers. To exclude heat transfer through the air, the cylinder was connected to a vacuum pump, which evacuated the cylinder to a pressure of about 5×15^{-5} mm mercury.

The strips were also made of stainless steel, 0·1 mm thick. The strip was clamped on one end in a copper block which was affixed to one of the covers and on the other side in a block which was connected with the second cover via a strong spring. This spring maintained a tension in the strip, even as it became longer at higher temperatures. The temperature of the strip T_1 was measured with five thermocouples which were spot welded to the strip. The temperature T_1 turns out to be uniform within 1°C. The thermocouple leads were all brought outside through glass insulating bushings in one cover plate. The strip was heated by alternating current via a regulating transformer. The electrical energy supplied can be calculated from the current through and the voltage difference across the strip. In the steady state this energy is equal to the net radiation transport ϕ_s, provided that there is no other sort of heat transfer from the strip. Heat transfer from the strip may take place,

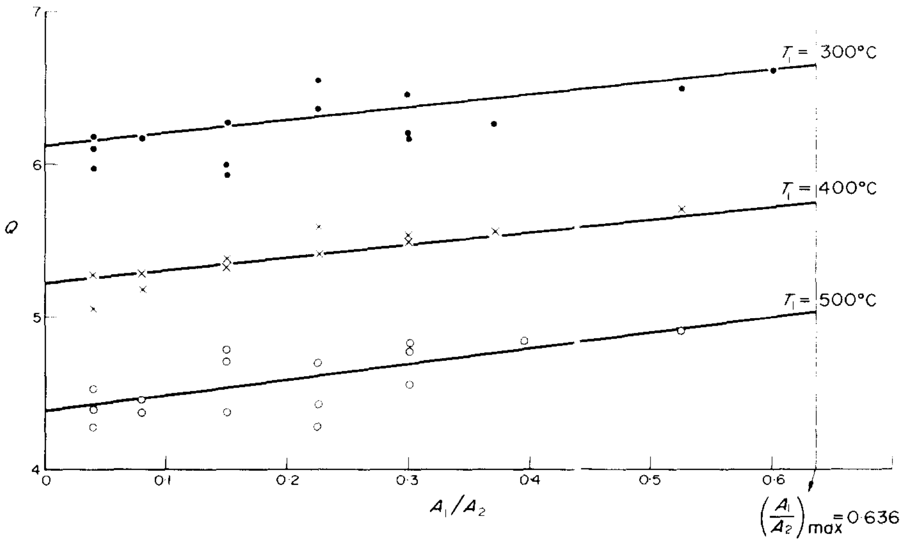

FIG. 3. The dependency of Q upon A_1/A_2 and T_1 at $\varepsilon = 0{\cdot}96$.

other than by radiation, by means of:

1. conduction via the thermocouple leads,
2. conduction and free convection to the air in the cylinder,
3. conduction through the strip to the cold covers.

It appeared that heat losses via the thermocouple leads and via the strip were negligibly small. The pressure in the cylinder is so low that the heat transfer through the air is also negligible. The net radiation transport is thus indeed equal to the electrical energy supplied.

RESULTS

Some results of the measurements are given in the Figs. 3–5. In these figures Q is shown as a

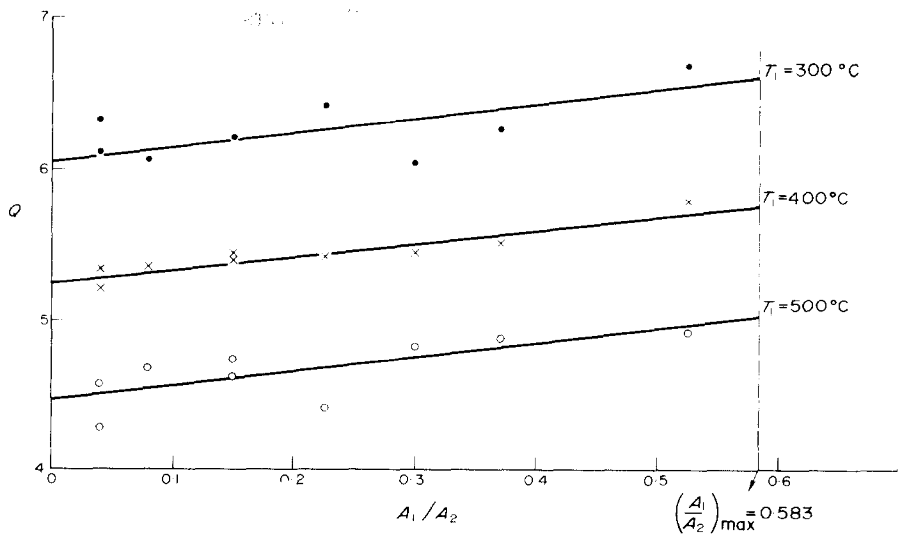

FIG. 4. The dependency of Q upon A_1/A_2 and T_1 at $\varepsilon = 0{\cdot}60$.

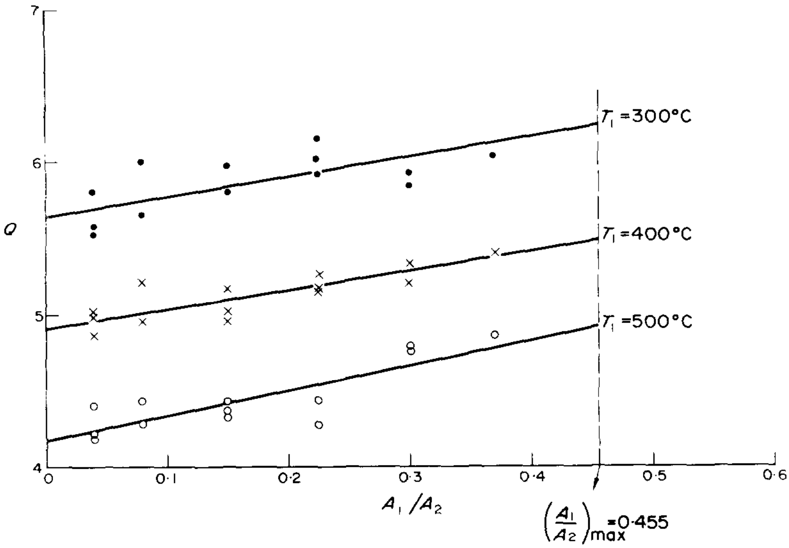

FIG. 5. The dependency of Q upon A_1/A_2 and T_1 at $\varepsilon = 0{\cdot}30$.

function of A_1/A_2 at T_1 equal to 300, 400 and 500°C; each of the three figures represents one value of ε, viz. 0·96, 0·60 and 0·30. The maximum possible value of A_1/A_2 (for the given ε) is also indicated in each figure*. In addition to these three values of ε, measurements were also made at $\varepsilon = 0{\cdot}9$, 0·75 and 0·45.

In all cases the relationship between Q and A_1/A_2 at constant ε and constant strip temperature appeared to be linear. This is in accordance with equation (8). The greatest deviation of a single point is about 5 per cent; a discrepancy well within the expected accuracy of the measurements: the relative, possible error is estimated to be ± 10 per cent.

The absorption coefficients are only a function of temperature, if the radiation surfaces can be considered as gray. If furthermore the simplified theory [equation (6)] is valid, the values of Q at any given temperature must yield the same function of A_1/A_2 for various values of ε. To check if the simplified theory with only temperature dependent absorption coefficients could be used to describe the experimental results, the values of a_1 and a_2 were calculated from each set of experiments representing Q as a function of A_1/A_2. The results are assembled in Table 2. It appears from these results that if for a constant strip temperature the absorption coefficients a_1 and a_2 are considered to be constant, independent

Table 2. Absorption coefficients as functions of T_1 *and* ε

T_1(°C)	ε	a_1	a_2	T_1(°C)	ε	a_1	a_2	T_1(°C)	ε	a_1	a_2
300	0·96	0·16	0·53	400	0·96	0·19	0·48	500	0·96	0·23	0·48
	0·90	0·16	0·51		0·90	0·19	0·50		0·90	0·23	0·52
	0·75	0·16	0·52		0·75	0·19	0·52		0·75	0·23	0·48
	0·60	0·17	0·51		0·60	0·19	0·54		0·60	0·22	0·52
	0·45	0·16	0·54		0·45	0·19	0·55		0·45	0·22	0·49
	0·30	0·17	0·44		0·30	0·20	0·44		0·30	0·24	0·38

* $(1 - \varepsilon)$ is the measure for the eccentricity; see Fig. 2.

of ε, and, hence, if the surfaces are treated as being gray, the simplified theory (6) holds even at high eccentricities ($0{\cdot}4 < \varepsilon < 1$). At $\varepsilon = 0{\cdot}3$ the value of a_2 is about 20 per cent lower than at higher ε's. In this region of eccentricity where the strip comes close to the cylinder wall, Q becomes apparently dependent on ε.

CONCLUSION

The experiments have produced the surprising result, that for the geometry considered, the measurements of net radiation transport match the simple theory of equation (6), assuming gray surfaces, even for high eccentricities ($0{\cdot}4 < \varepsilon < 1$). We expect that the simplifying assumptions underlying equation (6) are not so severe and that the engineering practice of using equation (6) also in situations which seem well outside the theoretical restrictions of this theory can be justified. Therefore, the authors feel that engineers who extensively use the equation (6) and try to do so consciously, would welcome a deeper analysis than that is already available in literature [1] as to how far its underlying restrictions can be relaxed.

REFERENCE

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INFLUENCE DE LA GÉOMÉTRIE SUR LE TRANSFERT THERMIQUE PAR RAYONNEMENT À L'INTÉRIEUR D'UN ESPACE CLOS.

Résumé—Dans la pratique de l'ingénieur, le flux thermique rayonné par des objets chauds est généralement calculé à partir de l'équation :

$$\phi_s = A_1\sigma(T_1^4 - T_2^4)\left[\frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right)\right]^{-1}$$

Cependant cette équation n'est valable seulement que si l'arrangement des surfaces A_1 et A_2 est complétement symétrique. Dans cette étude le transport par rayonnement est mesuré entre une bande chauffée et un cylindre mince refroidi, la largeur de la bande et la distance à la paroi du cylindre pouvant être variées. Bien que ces conditions soient fortement asymétriques, le transport par rayonnement est prédit par l'équation ci-dessus avec une précision acceptable pour l'ingénieur.

DER GEOMETRIEEINFLUSS AUF DEN WÄRMEAUSTAUSCH DURCH STRAHLUNG INNERHALB EINES GESCHLOSSENEN RAUMES

Zusammenfassung—In der Ingieurpraxis wird der Wärmestrom durch Strahlung von heissen Objekten zu ihrer Umgebung versuchsweise mit der Gleichung berechnet:

$$\phi_s = A_1\sigma(T_1^4 - T_2^4)\left[\frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right)\right]^{-1}$$

Diese Gleichung ist jedoch nur dann exakt, wenn die Anordnung der Oberflächen A_1 und A_2 vollkommen symmetrisch ist. In dieser Untersuchung wurde der Strahlungsstrom zwischen einem geheizten Band und einem schlanken, gekühlten Zylinder gemissen, wobei die Breite des Bandes und der Abstand des Bandes von der Zylinderwand variiert wurden.

Bei diesen Anordnungen, sogar bei dem hochgradig asymmetrischen, wurde der Strahlungsstrom durch die obige Gleichung mit annehmbarer Genauigkeit angegeben.

ВЛИЯНИЕ ГЕОМЕТРИИ НА ЛУЧИСТЫЙ ПЕРЕНОС ТЕПЛА В ЗАМКНУТОМ ПРОСТРАНСТВЕ

Аннотация—В инженерной практике скорость переноса тепла излучением от нагретых предметов в окружающую среду обычно рассчитывается по уравнению :

$$\phi_s = A_1\sigma(T_1^4 - T_2^4)\left[\frac{1}{a_1} + \frac{A_1}{A_2}\left(\frac{1}{a_2} - 1\right)\right]^{-1}$$

Однако, это уравнение явдяется точным только, если расположение поверхностей A_1 и A_2 полностью симметрично. В этой работе проводились измерения лучистого переноса между нагретой полоской и тонким охлажденным цилиндром, причем ширина полоски и расстояние между ней и стенкой цилиндра менялись.

В таких условиях, даже если поверхности расположены совершенно несимметрично, лучистый перенос рассчитывался с допустимой для пратических целей погрешностью с помощью данного уравнения.

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